4 In a class of 40 students, 19 play tennis, 20 play netball and 8 play neither of these sports. Determine the number of students in the class who:

a play tennis
b do not play netball
c play at least one of the sports
d play one and only one of the sports
e play netball, but not tennis

5 In a class of 25 students, 15 play hockey and 16 play basketball. If there are 4 students who play neither sport, determine the number of students who play both hockey and basketball.

6 In a class of 40, 34 like bananas, 22 like pineapples and 2 dislike both fruits. Find the number of students who:

a like both fruits
b like at least one fruit

7 In a group of 50 students, 40 study Mathematics, 32 study Physics and each student studies at least one of these subjects. From a Venn diagram find how many students:

a study both subjects
b study Mathematics but not Physics

8 In a class of 40 students, 23 have dark hair, 18 have brown eyes, and 26 have dark hair, brown eyes or both. How many students have:

a dark hair and brown eyes
b neither dark hair nor brown eyes
c dark hair but not brown eyes?

9 400 families were surveyed. It was found that 90% had a TV set and 60% had a computer. Every family had at least one of these items. How many of the families had both a TV set and a computer?

10 In a certain town 3 newspapers are published. 20% of the population read A, 16% read B, 14% read C, 8% read A and B, 5% read A and C, 4% read B and C and 2% read all 3 newspapers. What percentage of the population read:

a none of the papers
b at least one of the papers
c exactly one of the papers
d either A or B
e A only?

F MORE DIFFICULT PROBLEM SOLVING (EXTENSION)

EXERCISE 1F

1 In a circle of music lovers, 14 people play the piano or violin, 8 people are piano players, and 5 people play both instruments. Find the number of violin players.

2 Our team scored well in the interschool athletics. Eight of us gained places in running events, 5 gained places in both running and jumping events, and 14 of us collected exactly one place in running or jumping. How many places were gained by our team?

3 64% of students at a school study a language, 79% study mathematics and each student studies at least one of these subjects. What percentage of students study both a language and mathematics?

4 A survey is made of the investments of the members of a club. All of the 133 members own at least one type of share, 96 members owning mining shares, 70 having oil shares and 66 members having industrial shares. Of those who own mining shares, 40 also own oil shares and 45 also have industrial shares, while the number of those who own oil shares and industrial shares is 28. How many members of the club own all three types of shares?

5 At a certain school there are 90 students studying for their matriculation certificate. They are required to study at least one of the subjects: Physics, French and History. Of these students, 50 are studying Physics, 60 are studying French and 55 are studying History. Thirty students are studying both Physics and French, while 10 students are studying both French and History but not Physics. Twenty students are studying all three subjects. Construct and explain a Venn diagram which represents this situation.
Use this diagram to determine:

a how many students are studying both Physics and History, but not French

b how many students are studying at least two of these three subjects.

6 In a school of 405 pupils, a survey on sporting activities shows that 251 pupils play tennis, 157 play hockey and 111 play softball. There are 45 pupils who play both tennis and hockey, 60 who play hockey and softball and 39 who play tennis and softball. What conclusion may be drawn about the number of those who participate in all three sports?

G THE ALGEBRA OF SETS (EXTENSION)

For the set of real numbers $\mathbb{R}$, under the operations $+$ and $\times$, you should be aware of the following laws for real numbers a, b and c:

- **commutative** $a+b=b+a$ and $ab=ba$.
- **identity** Identity elements 0 and 1 exist such that, $a+0=0+a=a$ and $a\times 1=1\times a=a$.
- **associativity** $(a+b)+c=a+(b+c)$ and $(ab)c=a(bc)$
- **distributive** $a(b+c)=ab+ac$

In **Exercise 1D** we used a Venn diagram to verify that: $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$.

Notice that these results look very much like the *distributive* law for real numbers under $+$ and $\times$, with real numbers replaced by sets, $\times$ replaced by $\cup$ and $+$ replaced by $\cap$.

The following are the **laws for the algebra of sets** under the operations of $\cup$ and $\cap$.

- **commutative** $A\cap B=B\cap A$ and $A\cup B=B\cup A$
- **associativity** $A\cap(B\cap C)=(A\cap B)\cap C$ and $A\cup(B\cup C)=(A\cup B)\cup C$
- **idempotent** $A\cap A=A$ and $A\cup A=A$
- **distributive** $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ and $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$
- **DeMorgans** $A\cap\varnothing=\varnothing$, $A\cup U=U$, $(A\cap B)'=A'\cup B'$, $(A\cup B)'=A'\cap B'$
- **Complement** $(A')'=A$