

Year End Review: Combinatorics and Probability (Units 2 & 3)

A permutation is an arrangement of objects where order is important (like people or arranging letters). A combination is a selection of objects in which order is not important (like card games and 6-49).

Example 1: Determine the number of ways 6 people can be arranged in a line.

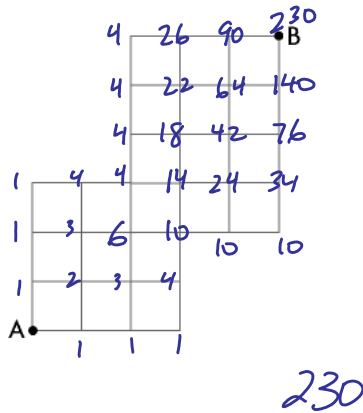
$$6! = 720$$

$$(or \ 6P_6)$$

Example 2: Determine the number of ways to arrange the letters in MISSIONS.

$$\frac{8!}{2!3!} = 3360$$

Don't forget about the number of pathways. Determine how many ways there are to go from point A to point B if you can only travel east or north.



The probability that an event will occur is the number of favourable outcomes divided by the total outcomes.

Example 3: Determine the probability of rolling a 5 on a dice and then tossing a tails with a coin.

$$\left(\frac{1}{6}\right)\left(\frac{1}{2}\right) = \frac{1}{12}$$

Example 4: If there are 6 white marbles and 4 black marbles in a bag, determine the probability of first choosing a black marble and then a white marble.

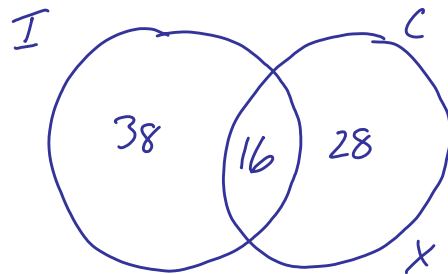
$$\frac{(4C_1)(6C_1)}{10C_2} = 0.53$$

Example 5: Determine the odds in favor and odds against drawing a king out of a deck of cards.

$$\begin{aligned} \text{in favor} &\Rightarrow 4:48 \text{ or } 1:12 \\ \text{against} &\Rightarrow 48:4 \text{ or } 12:1 \end{aligned}$$

A Venn diagram uses overlapping circles inside a rectangle to model statements. If two circles do not overlap, they are called mutually exclusive events and have no common outcomes.

Example 6: Students at Abby were surveyed and 54% had Internet access at home, 44% have cell phones and 16% have both. What is the probability that a randomly chosen student from Abby will have neither Internet access at home nor cell phones?



$$\begin{aligned} 100 &= 38 + 16 + 28 + x \\ 100 &= 82 + x \\ \underline{18} &= x \end{aligned}$$

If huge numbers of possibilities exist, we use combinatorics to help find probabilities.

Example 7: What is the probability of being dealt four 9's in a cribbage hand of 6 cards?

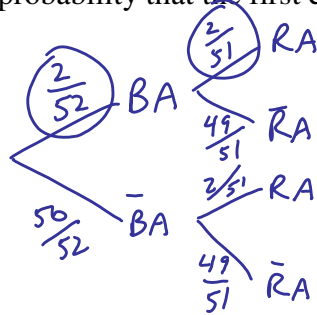
$$\frac{(4C_4)(48C_2)}{52C_6} = 0.0000554$$

Example 8: What is the probability of guessing the correct entry code if there are three numbers, from 0-9, and there are no repeated digits in the code?

$$\frac{1}{10C_3} = 0.0083$$

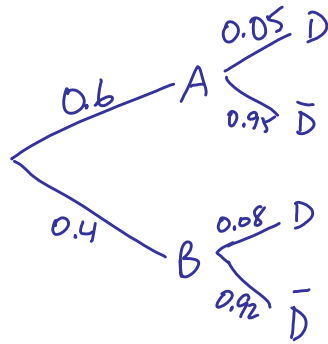
Conditional probability deals with probabilities of an event B, given that event A has occurred. Tree diagrams will help with these questions!!

Example 9: Two cards are drawn without replacement from a shuffled deck of 52 cards. What is the probability that the first card is a black ace and the second card is a red ace?



$$\left(\frac{2}{52}\right)\left(\frac{2}{51}\right) = 0.0015$$

Example 10: Two machines A and B produce all the items in a factory. Machine A produces 60% of the output. The percentages of defective items produced by these machines are 5% and 8% respectively. Determine the probability that an item was produced by machine A if it is known to be defective?



$$\begin{aligned} P(A|D) &= \frac{P(A \text{ and } D)}{P(D)} \\ &= \frac{(0.6)(0.05)}{(0.05)(0.6) + (0.08)(0.4)} \\ &= 0.484 \end{aligned}$$