Year End Review: Polynomial, Exponential, and Logarithmic Functions (Units 6 & 7)

A polynomial function in one variable is a function that contains only the operations of multiplication and addition, with real-number coefficients, whole-number exponents, and two variables. The degree of the function is the greatest exponent of the function. For example, $f(x) = 6x^3 + 3x^2 - 4x + 9$ is a polynomial function of degree 3. More specifically, it is a cubic function due to its degree is 3.

The graphs of polynomial functions of the same degree have common characteristics.

The end behaviour of a graph is the description of the shape of the graph, from left to right, on the coordinate plane. Cartesian grids are divided into four quadrants by the x-axis and y-axis.

Any point where the graph of a function changes from increasing to decreasing or from decreasing to increasing is called a turning point.

The domain is the set of all x values of a graph while the range of a function is the set of all y values of a graph.

The x-intercepts are where the graph crosses the x-axis and the y-intercepts are where the graph crosses the y-axis.

Example 1: Complete the following chart:

Function	**$f(x) = 3x + 2$**	**$g(x) = x^3 + x^2 + 2x - 2$**
Degree	1	3
Number of x-intercepts	1	1
Y-intercept	2	-2
End Behaviour	III → I	III → I
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$
Range	$y \in \mathbb{R}$	$y \in \mathbb{R}$
Number of Turning Points	0	2

An exponential function is of the form $y = a(b)^x$ where $a \neq 0$, $b > 0$, and $b \neq 1$. The graphs of exponential function are very unique.

The function $y = \log_{10}x$ is equivalent to $x = 10^y$, so a logarithm is an exponent. The meaning of $\log_{10}x$ is "the exponent that must be applied to base 10 to get the value of x". For example, $\log_{10}100 = 2$.

The expression $\log_{10}x$ is known as the common logarithm or a logarithm with a base of 10. The expression is often written without the 10, so the two functions $y = \log_{10}x$ and $y = \log x$ are equivalent.

The symbol e is a constant known as Euler's number. It is an irrational number that equals 2.718…. A logarithm with base e is called the natural logarithm and is written as lnx.

Example 2: Complete the following chart.

Function	**$y = 5(2)^x$**	**$y = 4(\frac{1}{2})^x$**	**$y = -4\log x$**	**$y = 13\ln x$**
Number of x-Intercepts	0	0	1	1
Y-intercept	5	4	none	none
End Behaviour	II → I	II → I	I → IV	IV → I
Domain	$x \in \mathbb{R}$	$x \in \mathbb{R}$	$x > 0$	$x > 0$
Range	$y > 0$	$y > 0$	$y \in \mathbb{R}$	$y \in \mathbb{R}$
Increasing/ Decreasing	inc	dec	dec	inc

When determining an equation that best fits the data, a graphing calculator must be used.

Example 3: Determine the equation of the exponential regression function of the following data.

x	~~0~~	1	2	3	4
y	~~0.0~~	2.1	4.2	6.3	8.4

$y = 1.48(1.58)^x$