Lesson Notes 7-9 **Pascal's Triangle & the Binomial Expansion**

Now we will look at a famous mathematical pattern know as Pascal's triangles.

$$
\begin{array}{ccccccccc}
 & & & & 1 & & & & \\
 & & & 1 & & 1 & & & \\
 & & 1 & & 2 & & 1 & & \\
 & 1 & & 3 & & 3 & & 1 & \\
1 & & 4 & & 6 & & 4 & & 1
\end{array}
$$

These numbers can also be found using **combinations**, or the ${}_nC_r$ function on the GDC.

${}_4C_0 = 1$ ${}_4C_1 = 4$ ${}_4C_2 = 6$ ${}_4C_3 = 4$ ${}_4C_4 = 1$

$\binom{n}{r}$, or ${}_nC_r$ represents the number of ways n items can be taken r at a time.

The number of combinations of n items taken r at a time is found by:

$$\frac{n!}{r!(n-r)!}, \text{ where } n! = n \times (n-1) \times (n-2) \times \ldots \times 1$$

Example 1: Find the value of $\binom{10}{5}$ using the formula, and check with your GDC.

$= \frac{10!}{5!(10-5)!} = 252$

Investigation – Patterns in Polynomials
Expand each of the following expressions.

1. $(a + b)^1$

 $a+b$

2. $(a + b)^2$

 $a^2+2ab+b^2$

3. $(a + b)^3$

 $a^3+3a^2b+3ab^2+b^3$

4. $(a + b)^4$

 $a^4+4a^3b+6a^2b^2+4ab^3+b^4$

Do you notice any similarities to Pascal's triangle? Based on these patternsn, predict what the expansion of $(a + b)^7$ might be. Coefficients are from Pascal's triangle.

$a^7+7a^6b+21a^5b^2+35a^4b^3+35a^3b^4+21a^2b^5+7ab^6+b^7$

The binomial theorem states that for any power of a binomial, where n ε N,

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n}a^0 b^n$$

Example 2: Use the binomial theorem to expand $(x + 4)^4$. Write your answer in simplest form.

$$= \binom{4}{0}x^4 4^0 + \binom{4}{1}x^3 \cdot 4^1 + \binom{4}{2}x^2 \cdot 4^2 + \binom{4}{3}x^1 \cdot 4^3 + \binom{4}{4}x^0 \cdot 4^4$$

$$= x^4 + 16x^3 + 96x^2 + 256x + 256$$

Example 3: Use the binomial theorem to expand $(3x – 5y)^3$. Write your answer in simplest form.

$$= \binom{3}{0}(3x)^3(-5y)^0 + \binom{3}{1}(3x)^2(-5y)^1 + \binom{3}{2}(3x)^1(-5y)^2 + \binom{3}{3}(3x)^0(-5y)^3$$

$$= 27x^3 - 135x^2y + 225xy^2 - 125y^3$$

Example 4: Find the x^3 term in the expansion of $(5x – 2)^9$.

$$\binom{9}{6}(5x)^3(-2)^6$$

$$= (84)(125x^3)(64)$$

$$= 672000x^3$$

Example 5: In the expansion of $(2x + 1)^n$, the coefficient of the x^3 term is 80. Find the value of n.

$$\binom{n}{3}(2x)^3(1^{n-3}) = 80x^3$$

$$\frac{n!}{3!(n-3)!}(8x^3)(1) = 80x^3$$

$$\frac{n(n-1)(n-2)\cancel{(n-3)}\ldots}{6\cancel{(n-3)(n-4)}\ldots}(8x^3) = 80x^3$$

$$n(n-1)(n-2) = 60$$

From GDC zeros $\Rightarrow n = 5$