Notes: Graphing $y = ax^2 + bx + c$ by completing the square (LT)

If a parabola is given in it's standard form: $y = a(x - p)^2 + q$, it is fairly easy to sketch, as we did in the previous lesson.
Sometimes, though, the parabola will be given in general form: $y = ax^2 + bx + c$, and then we'll need to change it into it's standard form by completing the square.

Prerequisite skills:
Expand these binomial squares:

$(x + 1)^2$ $(x+1)(x+1)$ $x^2+1x+1x+1 = x^2+2x+1$

$(x - 3)^2$ $(x-3)(x-3)$ $x^2-3x-3x+9 = x^2-6x+9$

$(x + 5)^2$ $(x+5)(x+5)$ $x^2+25+10x$ $x^2+5x+5x+25$

$(x + c)^2$ $(x+c)(x+c)$ $x^2+cx+cx+c^2 = x^2+2cx+c^2$

Fill in the blanks:

$x^2 - 14x + \underline{49} = (x - \underline{7})^2$ ($-14 \div 2$)

$x^2 + 20x + \underline{100} = (x + \underline{10})^2$

$x^2 - 22x + \underline{121} = (x - \underline{11})^2$

$x^2 + 7x + \underline{12.25} = (x + \underline{3.5})^2$ (3.5^2)

Write each equation in the form $f(x) = a(x - p)^2 + q$, and determine the vertex:

1. $y = (x^2 - 4x) + 11$ (-2^2)

$y = (x^2 - 4x + 4 - 4) + 11$

$y = (x^2 - 4x + 4) + 11 - 4$

$y = (x - 2)^2 + 7$ V: (2, 7)

min (+)

min/max point?

2. $y = (x^2 + 10x) - 3$ (5^2)

$y = (x^2 + 10x + 25 - 25) - 3$

$y = (x^2 + 10x + 25) - 3 - 25$

$y = (x + 5)^2 - 28$ V: (-5, -28)

min/max point?

3. $y = -(x^2 - 14x) - 6$

$y = -(x^2 - 14x + 49 - 49) - 6$

$y = -(x^2 - 14x + 49) - 6 - 49(-1)$

$y = -(x - 7)^2 + 43$ V: (7, 43)

max (-)

min/max point?

$y = 2x^2 - 8x + 1$

$y = 2(x^2 - 4x + 4 - 4) + 1$

$y = 2(x^2 - 4x + 4) + 1 - 4(2)$

$y = 2(x - 2)^2 - 7$ V: (2, -7)

min/max point?

5. $y = 3x^2 + 12x + 7$

$y = 3(x^2 + 4x + 4 - 4) + 7$

$y = 3(x^2 + 4x + 4) + 7 - 4(3)$

$y = 3(x + 2)^2 - 5$ V: (-2, -5)

min/max point?

6. $y = -4x^2 - 12x - 5$

$y = -4(x^2 + 3x + 2.25) - 5 - 2.25(-4)$

$y = -4(x + 1.5)^2 + 4$ V: (-1.5, 4)

min/max point?

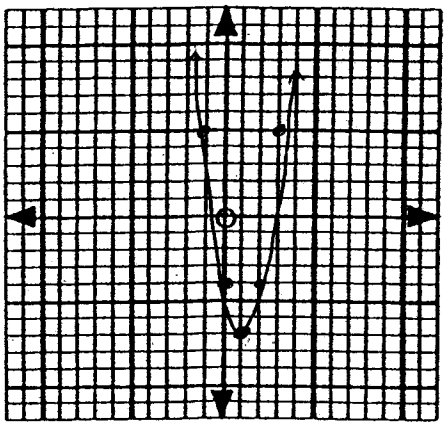
Complete the squares and then sketch these parabolas, labeling the vertex, the axis of symmetry, and any two other points on the graph:

1. $y = 3x^2 - 6x - 4$

$y = 3(x^2 - 2x + 1 - 1) - 4$

$y = 3(x^2 - 2x + 1) - 4 - 1(3)$

$y = 3(x-1)^2 - 7$

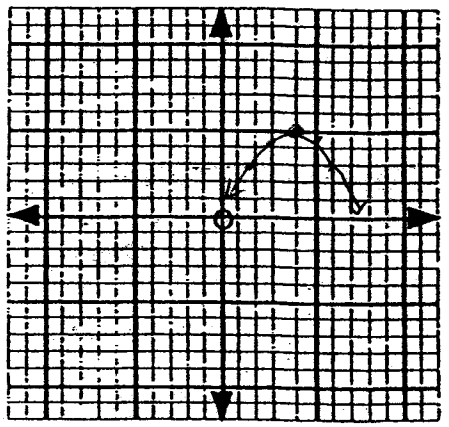


2. $y = -\frac{1}{2}x^2 + 4x - 3$

$y = \frac{-1}{2}(x^2 - 8x + 16 - 16) - 3$

$y = \frac{-1}{2}(x^2 - 8x + 16) - 3 - 16(\frac{-1}{2})$

$y = \frac{-1}{2}(x-4)^2 + 5$



Write an equation of a quadratic function that satisfies each of these conditions:

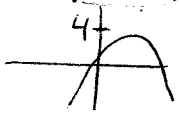
$y = a(x-p)^2 + 8$

1. The function has a minimum value of 8 at $x = 5$: $y = (x-5)^2 + 8$

2. The function has a maximum value of -3 at $x = -1$: $y = -(x+1)^2 - 3$

3. The parabola is congruent to $y = \frac{1}{4}x^2$, and has minimum value of -4: $y = \frac{3}{4}(x-0)^2 - 4$

4. The function has axis of symmetry $x = -8$, and it's range is $y \leq 4$: $y = -(x+8)^2 + 4$



5. The parabola has vertex $(3, -5)$ and point $(1, 1)$: $y = \frac{3}{2}(x-3)^2 - 5$

$y = a(x-3)^2 - 5$

$1 = a(1-3)^2 - 5$

$+5 \quad +5$

$\frac{6}{4} = \frac{a(4)}{4}$

$a = \frac{6}{4} = \frac{3}{2}$