

Solving Logarithmic Equations

Equations that contain logarithms, either common (base 10) or natural (base  $e$ ) can be solved algebraically or graphically. In all cases, though, we must be mindful of the restrictions on logarithms:

If  $y = \log_b a$ , then  $y \in \mathbb{R}$ ,  $b > 0$ ,  $b \neq 1$ ,  $a > 0$

1. Move all logarithms to one side, then convert into exponential form

a)  $\log_6(2x+1) = 1$       $2x+1=6^1$       $x = \frac{5}{2}$   
 $2x+1=6$   
 $2x=5$

b)  $\ln(2x) = 2$       $\frac{2x}{2} = \frac{e^2}{2}$       $x = 3.69$   
 $x = \frac{e^2}{2}$

c)  $\log_5(x-6) + \log_5(x-2) = 1$   
 $\log_5[(x-6)(x-2)] = 1$       $x^2 - 8x + 7 = 0$   
 $x^2 - 8x + 12 = 5^1$       $(x-7)(x-1) = 0$   
 $x = 7, x = 1$

d)  $\log(x) = 2 - \log(6)$   
 $\log(x) + \log 6 = 2$       $6x = 100$   
 $\log 6x = 2$       $x = 16.67$   
 $6x = 10^2$

e)  $\log_3(x^2) - \log_3(2x) = 2$   
 $\log_3\left(\frac{x^2}{2x}\right) = 2$       $\frac{x}{2} = 9$   
 $\log_3\left(\frac{x}{2}\right) = 2$       $x = 18$

f)  $\log_x(54) - \log_x(2) = 3$   
 $\log_x 27 = 3$       $\sqrt[3]{27} = \sqrt[3]{x^3}$   
 $3 = x$



2. Arrange so there is a single log on both side, then cancel both logs

a)  $2 \log(x) = \log(36)$

$\log(x^2) = \log 36$   $x = \pm 6$   
 $x^2 = 36$   $x = 6$

b)  $\ln(x) = \ln(3) - \ln(8)$

$\ln x = \ln \frac{3}{8}$   
 $x = \frac{3}{8}$

c)  $\log(2x+1) = 1 + \log(x-2)$

$\log(2x+1) = \log 10 + \log(x-2)$   $-8x = -21$   
 $\log(2x+1) = \log(10(x-2))$   $x = \frac{21}{8} (= 2.65)$   
 $2x+1 = 10x-20$

d)  $\log_{0.25}(x-4) + \log_{0.25}(x) = \log_{0.25}(5)$

$\log_{0.25}[x(x-4)] = \log_{0.25} 5$   $(x-5)(x+1) = 0$   
 $x^2 - 4x = 5$   $x = 5, -1$   
 $x^2 - 4x - 5 = 0$

3. When the bases are different, use your graphing calculator

a)  $2 \log_5(x) + 3 \log(x) = 10$

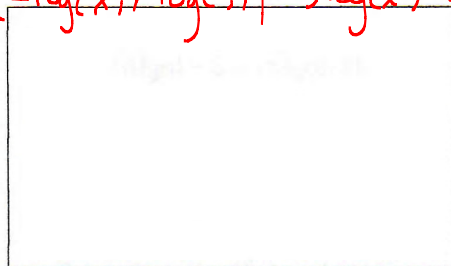
$y_1 = 2 \log_5(x) + 3 \log(x) - 10$

Restrictions:  $x > 0$

Asymptote:  $x =$  \_\_\_\_\_

Solution:  $x = 50.8$

$(2 \log(x) / \log(5)) + 3 \log(x) - 10$



b)  $\log_2(x) + \log_3(x+2) = 3$

$y_1 =$  \_\_\_\_\_

Restrictions: \_\_\_\_\_

Asymptote:  $x =$  \_\_\_\_\_

Solution:  $x =$  \_\_\_\_\_

