Lesson Notes PM 12 - 2.7

Solving Logarithmic Equations

Equations that contain logarithms, either common (base 10) or natural (base e) can be solved algebraically or graphically. In all cases, though, we must be mindful of the restrictions on logarithms:

$$\boxed{\text{If } y=\log_b a\text{, then } y\in\mathbb{R},\ b>0,\ b\neq 1,\ a>0}$$

1. Move all logarithms to one side, then convert into exponential form

a) $\log_6(2x+1)=1$

$2x+1=6^1$

$2x+1=6$

$2x=5$

$x=5/2$

b) $\ln(2x)=2$

$\frac{2x}{2}=\frac{e^2}{2}$

$x=\frac{e^2}{2}$

$x=3.69$

c) $\log_5(x-6)+\log_5(x-2)=1$

$\log_5[(x-6)(x-2)]=1$

$x^2-8x+12=5^1$

$x^2-8x+7=0$

$(x-7)(x-1)=0$

$x=7,\cancel{1}$

d) $\log(x)=2-\log(6)$

$\log(x)+\log 6=2$

$\log 6x=2$

$6x=10^2$

$6x=100$

$x=16.67$

e) $\log_3(x^2)-\log_3(2x)=2$

$\log_3\left(\frac{x^2}{2x}\right)=2$

$\log_3\left(\frac{x}{2}\right)=2$

$\frac{x}{2}=9$

$x=18$

f) $\log_x(54)-\log_x(2)=3$

$\log_x 27=3$

$\sqrt[3]{27}=\sqrt[3]{x^3}$

$3=x$

2. Arrange so there is a single log on both side, then cancel both logs

a) $2\log(x) = \log(36)$

$\log(x^2) = \log 36$ $\quad x = \pm 6$

$x^2 = 36$ $\quad x = 6$

b) $\ln(x) = \ln(3) - \ln(8)$

$\ln x = \ln 3/8$

$x = 3/8$

c) $\log(2x+1) = 1 + \log(x-2)$

$\log(2x+1) = \log 10 + \log(x-2)$ $\quad -8x = -21$

$\log(2x+1) = \log(10(x-2))$ $\quad x = 21/8 \; (\approx 2.65)$

$2x + 1 = 10x - 20$

d) $\log_{0.25}(x-4) + \log_{0.25}(x) = \log_{0.25}(5)$

$\log_{0.25}[x(x-4)] = \log_{0.25} 5$ $\quad (x-5)(x+1) = 0$

$x^2 - 4x = 5$ $\quad x = 5, \cancel{-1}$

$x^2 - 4x - 5 = 0$

3. When the bases are different, use your graphing calculator

a) $2\log_5(x) + 3\log(x) = 10$ $\quad (2\log(x)/\log(5)) + 3\log(x) - 10$

$y_1 = 2\log_5(x) + 3\log(x) - 10$

Restrictions: $x > 0$

Asymptote: $x =$ ______

Solution: $x =$ 50.8

b) $\log_2(x) + \log_3(x+2) = 3$

$y_1 =$ ______

Restrictions: ______

Asymptote: $x =$ ______

Solution: $x =$ ______