

Investigation – The Binomial Quiz

Here are five questions. The answer to each question is either 'true' or 'false'. Write down the answer to each question. You may need to guess the answer to some of them!

1. The bulletproof vest was created by a woman. *T*
2. On average, the 'eye muscles' move 300 000 times a day. *T*
3. Bowling was first played in Italy. *F*
4. It took Leonardo da Vinci ten days to paint Mona Lisa's lips. *F*
5. Selachophobia is the fear of light flashes. *F*

Now look at the answers at the back of the book to find the correct answers to these five questions. How many did you get right? Is this a good score? How many would you expect to get right if you were just guessing the answer to every question?

$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 2.5$$

To pass this 'test' you need to get 3 correct out of 5. What is the probability that you get exactly 3 right out of 5? *0.3125*

The essential elements of a binomial distribution are:

1. There is a fixed number of trials (n).
2. Each trial has only two possible outcomes - a "success" or a "failure".
3. The probability of a success (p) is constant from trial to trial.
4. Trials are independent of each other.

The outcomes of a **binomial experiment** and the corresponding probabilities of these outcomes are called a **binomial distribution**. The binomial distribution describes the behavior of a discrete variable X if the conditions above apply.

The parameters that define a unique binomial distribution are the values of n (the number of trials) and p (the probability of a success). Any binomial distribution is represented as $X \sim B(n, p)$.

Example 1: Determine the probability of getting exactly two heads in three tosses of a biased coin for which $P(\text{heads}) = 2/3$.

$$\begin{aligned} P(\text{two heads in three}) &= P(\text{HHT}) + P(\text{HTH}) + P(\text{THH}) \\ &= \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + \left(\frac{1}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) \\ &= 3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right) \\ &= \frac{12}{27} \\ &= \frac{4}{9} \end{aligned}$$

If X is binomially distributed, $X \sim B(n, p)$, then the probability of obtaining r successes out of n independent trials, when p is the probability of success for each trial is,

$$P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$$

This is often shortened to...

$$P(X = r) = \binom{n}{r} p^r q^{n-r} \quad \text{where } q = 1-p$$

Example 2: X is normally distributed with 7 trials and a probability of success equal to 0.25 at each attempt. Determine the probability of the following.

a. Exactly 5 successes

$$\begin{aligned} P(X=5) &= \binom{7}{5} (0.25)^5 (0.75)^{7-5} \\ &= 0.0115 \end{aligned}$$

b. At least one success.

$$\begin{aligned} P(X > 0) &= 1 - P(X=0) \\ &= 1 - \binom{7}{0} (0.25)^0 (0.75)^7 \\ &= 0.9999 \end{aligned}$$

c. Two or fewer successes.

$$\begin{aligned} P(X \leq 2) &= P(X=0) + P(X=1) + P(X=2) \\ &= \binom{7}{0} (0.25)^0 (0.75)^7 + \binom{7}{1} (0.25)^1 (0.75)^6 + \binom{7}{2} (0.25)^2 (0.75)^5 \\ &= 0.1335 + 0.3115 + 0.3115 = 0.7564 \end{aligned}$$

Example 3: The probability that I get a bus to work on any morning is 0.3. What is the probability that in a working week of five days I will get a bus only twice?

$$\begin{aligned} P(X=2) &= \binom{5}{2} (0.3)^2 (0.7)^3 \\ &= 0.309 \end{aligned}$$

Example 4: When administering a drug it was known that 80% of people using it were cured. The testing program administered the drug to two groups of 10 patients. What is the probability that all 10 patients were cured in both groups?

$$P(X=10) = \binom{10}{10} (0.8)^{10} (0.2)^0 \\ = 0.1074$$

$$\text{Two groups: } [P(X=10)][P(X=10)] \\ = 0.1074^2 \\ = 0.0115$$

Example 5: A box contains a large number of carnations of which one-quarter are red. The rest are white. Carnations are picked at random from the box. How many flowers must be picked so that the probability that there is at least one red carnation among them is greater than 0.95?

$$P(X \geq 1) > 0.95 \\ 1 - P(X=0) > 0.95 \\ 1 - 0.75^n > 0.95 \\ 0.05 < 0.75^n \\ \log 0.05 < \log 0.75^n \\ \log 0.05 < n \log 0.75 \\ \frac{\log 0.05}{\log 0.75} < n$$

$$n > 10.4$$

At least 11 carnations.