Solving Exponential Equations

An exponential equation has the variable in the exponent. In this lesson, we will see that logarithms are a useful, but not always necessary, tool to solve exponential equations.

Exponential Equation Type 1: Bases are equate-able

Solve for the variable by making the bases the same and then setting the adjusted exponents to be equal.

a) $6^{x-4} = 36$

$6^{x-4} = 6^2$

$x-4 = 2$

$x = 6$

b) $a^{4x} = a^{3-x}$

$4x = 3-x$

$5x = 3$

$x = \frac{3}{5}$

c) $\left(\frac{1}{3}\right)^{2x+2} = 27^x$

$3^{-1(2x+2)} = 3^{3x}$

$-2x-2 = 3x$

$-2 = 5x \quad x = -\frac{2}{5}$

d) $\sqrt[2]{7} = 49^{5x}$

$7^{\frac{1}{2}} = 7^{10x}$

$\frac{1}{2} = 10x$

$\frac{1}{10} = \frac{10x}{10}$

$\frac{1}{20} = x$

e) $64^{\frac{2}{x}} = 16^x$

$(4^3)^{\frac{2}{x}} = 4^{2x}$

$\frac{6}{x} = 2x$

$6 = 2x^2$

$\sqrt{3} = \sqrt{x^2}$

$x = \pm\sqrt{3}$

f) $\frac{2(36)^{2x-1}}{6^{2x}} = 72$

$\frac{36^{2x-1}}{6^{2x}} = 36$

$\frac{6^{2(2x-1)}}{6^{2x}} = 6^2$

$6^{4x-2-2x} = 6^2$

$4x-2-2x = 2$

$2x = 4$

$x = 2$

Exponential Equation Type 2: Bases are not equate-able, x in only one exponent

Solve for the variable by taking the log of both sides and using "$Exponent = \frac{\log(answer)}{\log(base)}$"

a) $6^{x-4} = 40$

$x-4 = \log_6 40$

$x = \log_6 40 + 4$

$x = 6.06$

b) $130 = 13^{2x}$

$\frac{\log_{13} 130}{2} = \frac{2x}{2}$

$0.949 = x$

c) $15(2)^{x+3} = 150$

$2^{x+3} = 10$

$x+3 = \log_2 10$

$x = \log_2 10 - 3$

$= 0.32$

Exponential Equation Type 3: Bases not equate-able, x in both exponents

Solve for the variable by taking the log of both sides, bringing the exponents down front.

a) $6^{x-4} = 75^x$

$\log 6^{x-4} = \log 75^x$

$(x-4)\log 6 = x\log 75$

$x\log 6 - 4\log 6 = x\log 75$

$-4\log 6 = x\log 75 - x\log 6$

$-4\log 6 = x(\log 75 - \log 6)$

$\frac{-4\log 6}{(\log 75 - \log 6)} = \frac{x(\log 75 - \log 6)}{\log 75 - \log 6}$

$x = -2.84$

b) $13^{2x+8} = 2^{x-1}$

$(2x+8)\log 13 = (x-1)\log 2$

$2x\log 13 + 8\log 13 = x\log 2 - \log 2$

$2x\log 13 - x\log 2 = -\log 2 - 8\log 13$

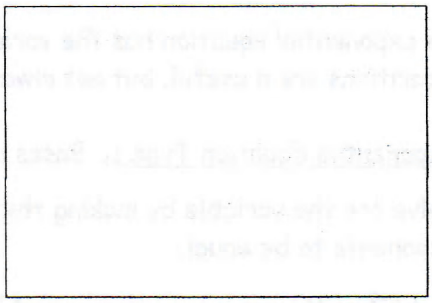
$\frac{x(2\log 13 - \log 2)}{2\log 13 - \log 2} = \frac{-\log 2 - 8\log 13}{2\log 13 - \log 2}$

$x = -4.78$

It is wise to remember that exponential equations can be solved using the graphing capabilities of your calculator. Let's graph one from each of the 3 types.

1. $64^{\frac{2}{x}} = 16^{x}$

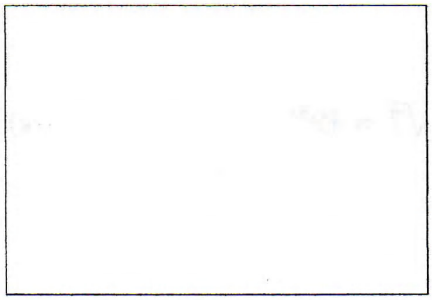
$y_1 =$ _____



X[,] Y[,]

2. $130 = 13^{2x}$

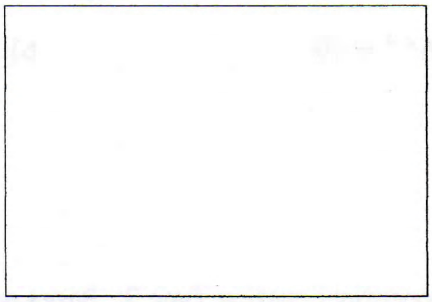
$y_1 =$ _____



X[,] Y[,]

3. $13^{2x+8} = 2^{x-1}$

$y_1 =$ _____



X[,] Y[,]