Lesson Notes 2-6 Solving Counting Problems

There are several different types of counting problems that we need to be able to solve.

**Example 1:** There are five swimmers in the first heat of a race: Aubrey, Betz, Cam, Deanna, and Elena. How many ways can the five swimmers finish first, second, and third?

$_5P_3 = 60$ $5 \times 4 \times 3$

**Example 2:** A teacher and her class are having a group photograph taken. There are 4 boys and six girls. The photographer wants the boys to sit together and the girls to sit together for one of the poses. How many ways can the students and teacher sit in a row of 11 chairs for this pose?

3 2 1

$4 \times 3 \times 2 \times 1$ $6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 1$

$= 4!6!1!3!$

$= 103{,}680$

A standard deck of 52 playing cars is summarized as follows:

Order is not important in the dealing of a group (hand) of cards. You usually rearrange then once they are in your hand.

| | Clubs | Diamonds | Hearts | Spades |
|---|---|---|---|---|
| Face Cards | K♣ | K♢ | K♡ | K♠ |
| | Q♣ | Q♢ | Q♡ | Q♠ |
| | J♣ | J♢ | J♡ | J♠ |
| | 10♣ | 10♢ | 10♡ | 10♠ |
| | 9♣ | 9♢ | 9♡ | 9♠ |
| | 8♣ | 8♢ | 8♡ | 8♠ |
| | 7♣ | 7♢ | 7♡ | 7♠ |
| | 6♣ | 6♢ | 6♡ | 6♠ |
| | 5♣ | 5♢ | 5♡ | 5♠ |
| | 4♣ | 4♢ | 4♡ | 4♠ |
| | 3♣ | 3♢ | 3♡ | 3♠ |
| | 2♣ | 2♢ | 2♡ | 2♠ |
| | A♣ | A♢ | A♡ | A♠ |
| | Black | Red | Red | Black |

**Example 4:** There are 12 women and 8 men interested in committee work.

a) Calculate the number of ways that a committee of 5 people can be chosen from this group if there must be exactly 3 women?

$$({}_{12}C_3)({}_8C_2) = 6160$$

b) Calculate the number of different groups of 5 that can be formed if there must be at least 3 women in each group.

3 W 4W 5W

$$({}_{12}C_3)({}_8C_2) + ({}_{12}C_4)({}_8C_1) + ({}_{12}C_5)({}_8C_0)$$

$$= 6160 + 3960 + 792$$

$$= 10912$$

**Example 5:** Solve for n: $4({}_nC_2) = 2({}_nC_1)$

$${}_nC_r = \frac{n!}{r!(n-r)!}$$

$$\frac{4n!}{2!(n-2)!} = \frac{2n!}{1!(n-1)!}$$

$$\frac{4n(n-1)\cancel{(n-2)(n-3)\ldots}}{2\cancel{(n-2)(n-3)(n-4)\ldots}} = \frac{2(n)\cancel{(n-1)(n-2)(n-3)\ldots}}{\cancel{(n-1)(n-2)(n-3)\ldots}}$$

$$2n(n-1) = 2n$$

$$2n^2 - 2n = 2n$$

$$2n^2 - 4n = 0$$

$$2n(n-2) = 0$$

$$2n = 0 \quad n-2 = 0$$

$$n = \cancel{0} \quad n = 2$$