

It is important to know the difference between a permutation and a **combination**. The two formulae are:

$${}_n P_r = \frac{n!}{(n-r)!} \qquad {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

A permutation is an arrangement of a set of objects where order is important.

A combination is a selection from a group of objects, where order is not important.

Example 1: With Lotto 6/49, there is a selection of 6 numbers from 49. The Lottery Corporation jumbles 49 numbered balls in a cage and allows 6 balls to drop out. The order in which the six numbers appears is not important. Eventually, the numbers are rearranged from lowest to highest. How many arrangements of 6 numbers are possible?

$${}_{49} C_6 = 13,983,816$$

Example 2: How many different groups of 4 people can be chosen from a group of 30?

$${}_{30} C_4 = 27405$$

Example 3: If the 4 people chosen were getting ^{titles} ~~titles~~: President, Vice-President, Treasurer, and Secretary, how many different groups can be formed from a group of 30 people?

$${}_{30} P_4 = 657,720$$

Example 4: There are 12 women and 8 men interested in committee work.

- a) Calculate the number of ways that a committee of 5 people can be chosen from this group if there must be exactly 3 women?

$$\binom{12}{3} \binom{8}{2} = 6160$$

- b) Calculate the number of different groups of 5 that can be formed if there must be at least 3 women in each group.

$$\begin{array}{ccc} \begin{array}{c} \underline{3W} \\ \binom{12}{3} \binom{8}{2} \\ 6160 \end{array} & + & \begin{array}{c} \underline{4W} \\ \binom{12}{4} \binom{8}{1} \\ 3960 \end{array} & + & \begin{array}{c} \underline{5W} \\ \binom{12}{5} \binom{8}{0} \\ 792 \end{array} \\ & & & & = 10912 \end{array}$$

Example 5: Solve for n: $4\binom{n}{2} = 2\binom{n}{1}$

$$\begin{aligned} \frac{4n!}{2!(n-2)!} &= \frac{2n!}{1!(n-1)!} \\ \frac{4n(n-1)(n-2)\dots}{2(n-2)(n-3)\dots} &= \frac{2n(n-1)(n-2)\dots}{(n-1)(n-2)(n-3)\dots} \\ 2n(n-1) &= 2n \\ 2n^2 - 2n &= 2n \\ 2n^2 - 4n &= 0 \\ 2n(n-2) &= 0 \\ \frac{2n}{2} = 0 & \quad \frac{n-2}{1} = 0 \\ n=0 & \quad n=2 \end{aligned}$$