In lesson 1, we determined the number of arrangements of objects when the objects were all different (ie. the letters in CLARINET were conveniently different).

Now, let's take any word that has two letters the same – ALL – and see how many different arrangements we can make. We'll start by differentiating the L's: AL_1L_2

AL_1L_2, ~~AL_2L_1~~, L_1AL_2, L_1L_2A, ~~L_2AL_1~~, ~~L_2L_1A~~

This is 3! = 6 "different" arrangements. Realistically, though, without the subscript numbers, AL_1L_2 is really the same as AL_2L_1 and if we eliminated all such similar arrangements, we'd be left with only 3 different arrangements: $\frac{3!}{2!} = 3$

If there are identical letters (or objects), we can still determine the number of different arrangements by dividing the original factorial by any (or all) identical objects.

Example 1: How many ways are there to arrange the letters in the following words:

a) BUBBLE (b) MISSING (c) MISSISSIPPI

$\frac{6!}{3!} = 120$ $\frac{7!}{2!2!} = 1260$ $\frac{11!}{4!4!2!} = 34650$

Example 2: How many 9-digit numbers can be formed using the numbers 1 1 2 2 2 3 3 3 3?

$$\frac{9!}{2!3!4!} = 1260$$

Example 3: How many ways can 4 red, 3 blue, and 6 green marbles be distributed among 13 children, if each child is to receive 1 marble?

$$\frac{13!}{4!3!6!} = 60060$$

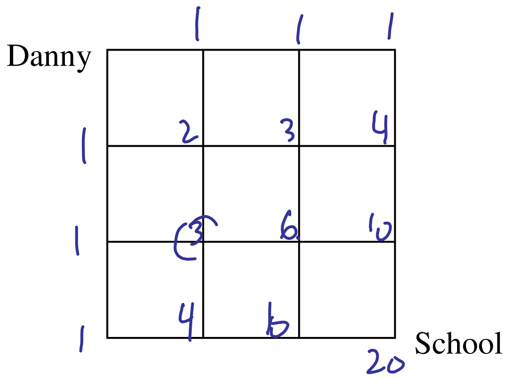
Example 4: To play Lotto 6/49, you must choose six numbers from 1 to 49. To win, all six of your numbers must be chosen. How many different Lotto 6/49 tickets are possible?

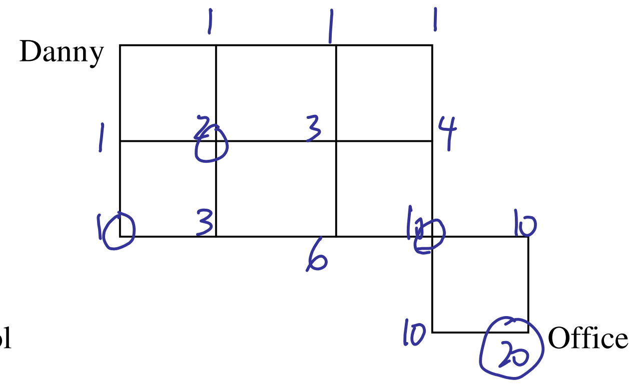
$${}_{49}P_6 = 1 \times 10^{10}$$

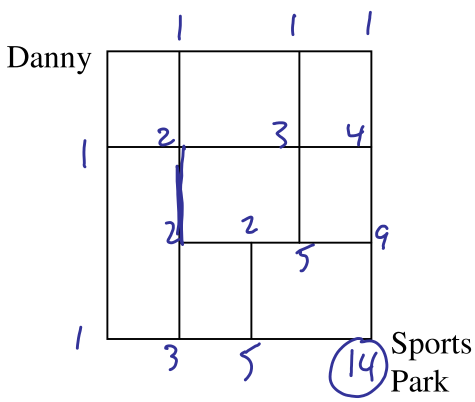
Pathways

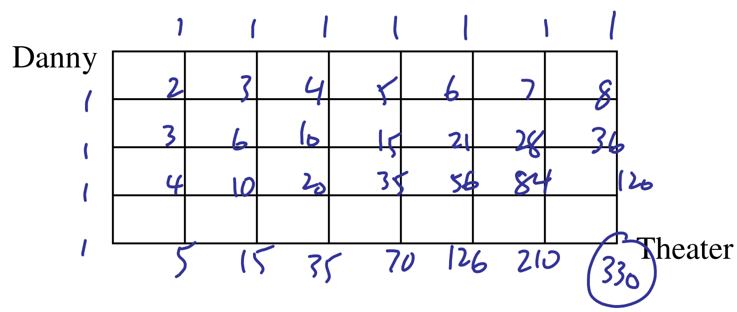
Determine the number of ways Danny can get to each of his destinations if he must travel along the given paths, can only travel east or south, and can not backtrack.

$$\frac{6!}{3!3!} = 20$$









Danny's trip to the theater is a complicated one, and one that can be determined without actually counting. He has to make a total of 11 direction decisions (but you can consider the 7 blocks east as identical and the 4 blocks south as identical). In other words, consider how many arrangements of EEEEEEESSSS there are.

This short cut will work for Danny's trip to school as well, but not for his trips to the office of the Sports Park because the pathways are a little more in these scenarios.