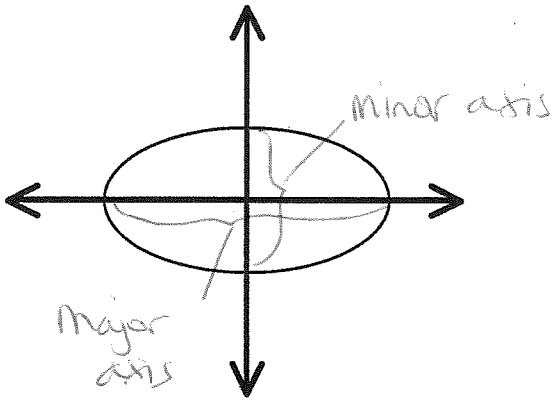
Lesson Notes 6-3 The Ellipse

An ellipse is an oval shape and has two axis of symmetries.



Each axis of symmetry intersects the ellipse in two places. The longer line segment between the intersection points of the ellipse and one axis of symmetry is the major axis.

The shorter line segment between the intersection points of the ellipse and the other axis of symmetry is the minor axis.

For an ellipse, half the length of the major axis is denoted by a and half the length of the minor axis by b. The major axis has length 2a units and the minor 2b units.

The standard form of an ellipse, centered at the origin with a major axis of length 2a along the x-axis and a minor axis of length 2b along the y-axis, is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

OR

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$$

The standard form of an ellipse, centered at any point (h, k) with a major axis of length 2a along the x-axis and a minor axis of length 2b along the y-axis, is

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

OR

$$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

The general form of an ellipse is found the same way as with circles.

Example 1: Write the equation for each ellipse in standard form and general form.

a) Center at (0, 0), ~~foci at (3, 0) and (-3, 0)~~, length of major axis 10, and length of minor axis 8.

in the x-axis direction

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$$

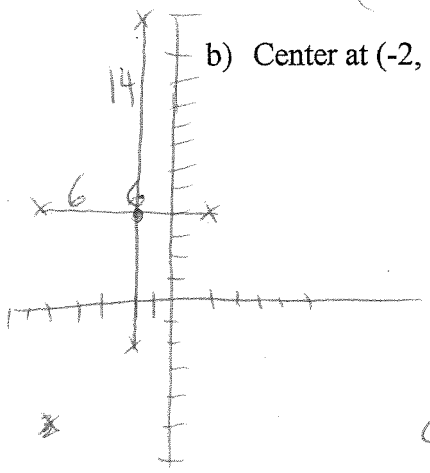
$$\frac{(x-0)^2}{5^2}+\frac{(y-0)^2}{4^2}=1$$

$$25\left(\frac{x^2}{25}+\frac{y^2}{16}=1\right) \leftarrow \text{standard form}$$

$$16\left(x^2+\frac{25y^2}{16}=25\right)$$

$$16x^2+25y^2=400$$

$$16x^2+25y^2-400=0$$



b) Center at (-2, 5), passing through (-5, 5), (1, 5), (-2, -2), and (-2, 12).

major axis = 14

minor axis = 6

$$\frac{(x+2)^2}{3^2}+\frac{(y-5)^2}{7^2}=1$$

$$9\left(\frac{(x+2)^2}{9}+\frac{(y-5)^2}{49}=1\right)$$

$$49\left((x+2)^2+\frac{9(y-5)^2}{49}=9\right)$$

$$49(x+2)^2+9(y-5)^2=441$$

$$49(x+2)^2+9(y-5)^2-441=0$$

Example 2: Find the coordinates of the center, the length of the major and minor axes, and the coordinates of the foci of each ellipse.

a) $x^2 + 2y^2 - 2x + 4y - 1 = 0$

$x^2 - 2x + 1 + 2y^2 + 4y = 1 + 1$

$x^2 - 2x + 1 + 2(y^2 + 2y + 1) = 2 + 1(2)$

$\frac{(x-1)^2}{4} + \frac{2(y+1)^2}{4} = \frac{4}{4}$

$\frac{(x-1)^2}{4} + \frac{(y+1)^2}{2} = 1$

Center: $(1, -1)$

Major: $\sqrt{4} = 2 \times 2 = 4$ (x-axis)

Minor: $\sqrt{2} = 1.41 \times 2 = 2.82$ (y-axis)

b) $3x^2 + y^2 + 6x - 8y - 11 = 0$

$3x^2 + 6x + y^2 - 8y = 11$

$3(x^2 + 2x + 1) + y^2 - 8y + 16 = 11 + 1(3) + 16$

$\frac{3(x+1)^2}{30} + \frac{(y-4)^2}{30} = \frac{30}{30}$

$\frac{(x+1)^2}{10} + \frac{(y-4)^2}{30} = 1$

C: $(-1, 4)$

Minor: $\sqrt{10} = 3.16 \times 2 = 6.32$

Major: $\sqrt{30} = 5.48 \times 2 = 10.96$