

The compound interest formula has several equivalent forms that you can use to solve compound interest problems.

To determine the future value, use $A = P(1 + i)^{nt}$.

To determine the present value, or principal, use $P = \frac{A}{(1 + i)^{nt}}$ $i = \frac{r}{n}$

To compare investments, use the rate of return, $\frac{A}{P} = (1 + i)^{nt}$

$$3000 = P \left(1 + \left(\frac{0.067}{2} \right)^n \right)^{nt}$$

Example 1: Ethan has won a valuable cash award in a science fair. He plans to invest some of the cash in an account that offers 6.7% interest, compounded semi-annually. He wants the investment to have a future value of \$3000 after 5 years. How much does Ethan need to invest now?

$$P = \frac{A}{\left(1 + \frac{r}{n} \right)^{tn}} = \frac{3000}{\left(1 + \frac{0.067}{2} \right)^{5 \cdot 2}} = \frac{3000}{1.3903} = 2157.81$$

Example 2: Emma has invested \$12 300 in a registered education savings plan (RESP). Emma wants her investment to grow to at least \$40 000, so that her newborn can go to university at age 18. What interest rate, compounded annually, will result in a future value of \$40 000?

$$A = P \left(1 + \frac{r}{n} \right)^{tn}$$

$$40000 = 12300 \left(1 + \frac{r}{1} \right)^{18 \cdot 1}$$

$$\frac{40000}{12300} = \frac{12300}{12300} \left(1 + r \right)^{18}$$

$$3.2520 = \sqrt[18]{(1+r)^{18}}$$

$$1.0677 = 1 + r$$

$$0.0677 = r \quad 6.8\% = r$$

Example 3: Wyatt has a choice between two investments.

- Investment A: A 10 year bond with an interest rate of 4.8%, compounded annually, and a future value of \$70 000
- Investment B: A 5 year bond with an interest rate of 4.8%, compounded annually, and a future value of \$35 000

Which investment has the greatest ratio of future value to present value? Explain briefly what this means.

$$\frac{A}{P} = \left(1 + \frac{r}{n} \right)^{tn}$$

Invest A $= \left(1 + \frac{0.048}{1} \right)^{10 \cdot 1} = 1.598 \Rightarrow \text{less } P$

Invest B $= \left(1 + \frac{0.048}{1} \right)^{5 \cdot 1} = 1.264 \Rightarrow \text{more } P$