The Laws of Logarithms

Recall that a logarithm is an exponent. Change the following from log form to exponential:

a) $\log_3 9 = 2$ $9 = 3^2$

c) $\log_x e = y$ $e = x^y$

b) $\log_{10} x = 5$ $x = 10^5$

d) $15 = \log_{0.5} x$ $0.5^{15} = x$

Change the following from exponential form to log form:

a) $5^3 = 125$ $\log_5 125 = 3$

c) $10^x = 1000$ $\log_{10} 1000 = x$

b) $a^x = y$ $\log_a y = x$

d) $15 = m^n$ $\log_m 15 = n$

Exponents have rules, remember? Let's practice them:

a) x^2x^6 x^8

b) $(3x^5)(4x^{-2})$ $12x^3$

c) $(4x^3)(-x^{10})$ $-4x^{13}$

d) $(x)^{-5}$ $\frac{1}{(x)^5}$

e) $4\cancel{(x^2y^4z^3)^0}$ 4

f) $\frac{5x^2y^{-3}}{25x^4y^5}$ $5x^2y^8$

g) $(x^3)^4$ x^{12}

h) $(2x^3y^5)^4$ $16x^{12}y^{20}$

i) $5(xy^3)^2$ $5x^2y^6$

j) $x^{\frac{1}{2}}$ $\sqrt[2]{x}^{\,1}$

k) $(x+3)^{\frac{2}{3}}$ $\sqrt[3]{(x+3)}^{\,2}$

l) $\frac{1}{\sqrt{5x}}$ $(5x)^{-1/2}$

Logarithms are exponents, so they also have rules.

Power Rule

$\log 2^3 =$ $\log(2\wedge 3) = 0.903$

$3\log 2 =$ 0.903

$\log_3 7^2 = \log(7^2) \div \log(3) =$ 3.542

$2\log 7 \div \log 3 =$ 3.542

$\boxed{\log_a x^n = n\log_a x}$

Restrictions: $a > 0,\ a \neq 1,\ x > 0$

This rule is the key to solving equations with a variable in the exponent!!

Example: Solve for x using logarithms: $5^x = 50$

$\log 5^x = \log 50$

$\frac{x\log 5}{\log 5} = \frac{\log 50}{\log 5}$

$x = 2.431$

Multiplication Rule

$\log(7 \times 3) =$ 1.322 $\qquad \log 7 + \log 3 =$ 1.322

$$\boxed{\log_a xy = \log_a x + \log_a y}$$

Restrictions: $x > 0,\ y > 0,\ a > 0,\ a \neq 1$

Examples: Simplify to a single log and then evaluate:

a) $\log_6 3 + \log_6 12 = \log_6(3 \times 12) = \log_6(36) = 2$

b) $\log_4 2 + \log_4 8 = \log_4(2 \times 8) = \log_4 16 = 2$

c) $\log_5 2 + \log_5 \frac{1}{10} = \log_5\left(2 \times \frac{1}{10}\right) = \log_5\left(\frac{1}{5}\right) = -1$

d) $\log 1 + \log 10 = \log(1 \times 10) = \log(10) = 1$

Quotient Rule

$\log\left(\frac{25}{5}\right) =$ 0.699 $\qquad \log 25 - \log 5 =$ 0.699

$$\boxed{\log_a\left(\tfrac{x}{y}\right) = \log_a x - \log_a y}$$

Restrictions: $x > 0,\ y > 0,\ a > 0,\ a \neq 1$

Examples: Simplify to single logarithms.

a) $\log a + 2\log b - \frac{1}{2}\log c = \log a + \log b^2 - \log c^{\frac{1}{2}} = \log\left(\frac{ab^2}{c^{\frac{1}{2}}}\right) = \log\left(\frac{ab^2}{\sqrt{c}}\right)$

b) $\log 500 - \log 5 = \log(100) = 2$

c) $\log_8 4 + \log_8 32 - \log_8 2 = \log_8(64) = 2$

If $\log 15 = x$, express each of the following in terms of x:

a) $\log 15^n = n \log 15 = nx$

b) $\log 150 = \log 15 + \log 10 = x + 1$

c) $\log 3 + \log 5 = \log 15 = x$

d) $\log \sqrt[3]{15} = \log 15^{1/3} = x^{1/3} = \sqrt[3]{x}$

e) $\log \frac{15}{1000} = \log 15 - \log 1000 = x - 3$

f) $\log 225 = \log 15 + \log 15 = x + x = 2x$

If $\log_6 x = 10$, express each of the following in terms of x:

a) $\log_6 6x =$ ________

b) $\log_6 x^3 =$ ________

c) $\log_6 \sqrt{x} =$ ________

d) $\log_6\left(\frac{36}{x}\right) =$ ________