Factorial notation is a concise representation of the product of consecutive descending natural numbers: $n! = n(n-1)(n-2)\ldots(3)(2)(1)$. For example, $4! = (4)(3)(2)(1)$.

Example 1: Evaluate the following.

a) $6! = 720$

(b) $\left(\dfrac{13!}{4!\,9!}\right) = 715$

Example 2: Simplify the following, where n is a natural number.

a) $(n+3)(n+2)!$

$(n+3)(n+2)(n+1)(n)(n-1)\ldots$

$= (n+3)!$

(b) $\dfrac{(n-3)!}{n!} = \dfrac{\cancel{(n-3)}\cancel{(n-4)}\cancel{(n-5)}\ldots}{n(n-1)(n-2)\cancel{(n-3)}\ldots}$

$= \dfrac{1}{n(n-1)(n-2)}$

Example 3: Solve the following equations given in factorial notation.

a) $\dfrac{n!}{(n-2)!} = 90$

$\dfrac{n(n-1)\cancel{(n-2)(n-3)\ldots}}{\cancel{(n-2)(n-3)(n-4)\ldots}}$

$n(n-1) = 90$

$n^2 - n = 90$

$n^2 - n - 90 = 0$

$(n+9)(n-10) = 0$

$n+9=0$; $n-10=0$

$\cancel{n=-9}$; $\boxed{n=10}$

(b) $\dfrac{(n+4)!}{(n+2)!} = 6$

$\dfrac{(n+4)(n+3)\cancel{(n+2)(n+1)\ldots}}{\cancel{(n+2)(n+1)(n)(n-1)\ldots}}$

$(n+4)(n+3) = 6$

$n^2 + 3n + 4n + 12 = 6$

$n^2 + 7n + 6 = 0$

$(n+6)(n+1) = 0$

$n = \cancel{-6}, \cancel{-1}$ no solution