Defining a Logarithm

Find the **LOG** button on your calculator and use it to compute the following:

a) $\log 1000 =$ 3

b) $\log 100 =$ 2

c) $\log 10 =$ 1

d) $\log 1 =$ 0

e) $\log 0 =$ —

f) $\log \frac{1}{10} =$ -1

g) $\log \frac{1}{100} =$ -2

h) $\log \frac{1}{1000} =$ -3

i) $\log 5 =$ 0.70

To better understand these **common logs** (base 10), we can change from log form to exponential form as follows:

$\log 1000 = 3$ can also be written as $\log_{10} 1000 = 3$ and we know that $10^3 = 1000$

We use base 10 so often that we call it the common log. We can, however, have different bases -- and if we do, we can quickly change the base back into base 10 as follows:

$$\log_3 9 = \frac{\log_{10}(9)}{\log_{10}(3)} = \log(9) \div \log(3) = 2 \qquad \text{(because } 3^2 = 9\text{)}$$

Use your calculator (or your algebra and exponent skills) to simplify the following:

a) $\log_2 8 =$ 3 $\log 8 \div \log 2$

b) $\log_3 27 =$ 3

c) $\log_1 15 =$ —

d) $\log_8(-10) =$ —

e) $\log_{(0.5)} 35 =$ -5.13

f) $\log_7 49 =$ 2

g) $\log_{36} 6 =$ 0.5

h) $\log_{(-2)} 4 =$ —

i) $\log_9 82 =$ 2.00

j) $\log_6(0.25) =$ -0.77

A logarithm is an exponent!! The base of the log is the base of the exponent!

$y = \log_{(b)}(a)$ <-------------------------> $b^y = a$

b is the base | **~~y~~ b is the base**

b is the argument | **a is the argument & stands alone**

Restrictions for logarithms:

Given: $y = \log_{(b)}(a)$

$$\boxed{y \in \mathbb{R},\ a > 0,\ b > 0 \text{ and } b \neq 0}$$

Determine the value of x in the following equations:

a) $\log_8 64 = x$ $64 = 8^x$ $8^2 = 8^x$ $x = 2$

b) $2 = \log_7 x$ $7^2 = x$ $x = 49$

c) $\log_{10} 10000 = x$ $10000 = 10^x$ $10^4 = 10^x$ $x = 4$

d) $\log_{(\sqrt{2})} x = 1$ $x = \sqrt{2}^1 = \sqrt{2}$

e) $\log_8 \sqrt{3} = x$ $\sqrt{3} = 8^x$ $x = 0.26$

f) $\log x = 2$

g) $\log_x 81 = 4$

h) $\log_{(\frac{1}{2})} x = 3$

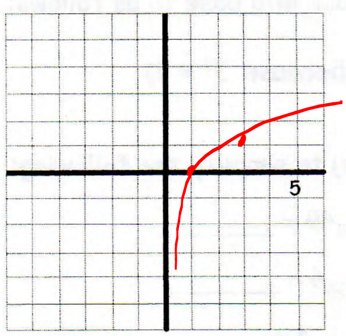
i) $\log_x 27 = \frac{1}{3}$

j) $\log_2 x = -3$

Graphing Logarithms

Look at the problem you run into if I ask you to graph: $x = 3^y$. To use your graphing calculator, you'll have to isolate y. Logarithms allow you to do that.

$x = 3^y$ <----------> $\log_3 x = y$ <----------> $y = \log(x) \div \log(3)$



Notice that the asymptote is vertical at: $x = 0$

Transformations work on logs, too. Given: $y = \log_3 x$,

a) move it 3 to the left: $y = \log_3(x+3)$ asymptote is at: $x = -3$

b) move it 5 up: $y = \log_3 x + 5$ asymptote is at: $x = 0$

c) move it 5 right, 2 down: $y = \log_3(x-5) - 2$ asymptote is at: $x = 5$

d) expand it vertically by a factor of 5, compress it horizontally by a factor of $\frac{1}{2}$:

$y = 5\log_3 2x$ asymptote is at: $x = 0$

$5\log(2x) \div \log 3$