Lesson Notes 6-2 The Circle

A circle is the set or locus of all points in a plane which are equidistant from a fixed point. This fixed point is called the center. The distance from this center to any point on the circle is called the radius.

The **standard form** of a circle with center (h, k) and radius r is

$$(x - h)^2 + (y - k)^2 = r^2$$

The **general form** of a circle can be obtained by expanding the standard form.

Example 1: Write the equation of the circle with center (-3, 5) and radius 6 in standard form and general form.

$(x-(-3))^2+(y-5)^2=6^2$

$(x+3)^2+(y-5)^2=36 \Leftarrow$ standard form

$(x+3)(x+3)+(y-5)(y-5)=36$

$x^2+3x+3x+9+y^2-5y-5y+25=36$

$x^2+6x+y^2-10y+34=36$

$x^2+6x+y^2-10y-2=0 \Leftarrow$ general form

Example 2: Write the equation of the circle with center (4, -1) and passing through (3, 7) in standard form and general form.

$(x-4)^2+(y+1)^2=(\sqrt{65})^2$

$(x-4)^2+(y+1)^2=65$

$x^2-8x+16+y^2+2y+1=65$

$x^2-8x+y^2+2y-48=0$

$d=\sqrt{(3-4)^2+(7--1)^2}$

$d=\sqrt{65}$

To determine the center of a circle written in general form, you must convert the equation into standard form. To do this you must complete the square. Completing the square allows you to factor the equation.

Example 3: Find the center and radius of the circle: $x^2 + y^2 + 14x - 72 = 0$.

$x^2 + 14x \boxed{+49} + y^2 = 72 + 49$ $\quad \frac{14}{2} = 7^2 = 49$

$(x+7)(x+7) + y^2 = 121$

$(x+7)^2 + y^2 = 121$

Center $= (-7, 0)$

radius $= \sqrt{121} = 11$

Example 4: Find the center and radius of the circle: $x^2 - 6x + y^2 - 8y - 39 = 0$.

$\frac{6}{2} = 3^2$

$x^2 - 6x + 9 + y^2 - 8y + 16 = 39 + 9 + 16$

$\Rightarrow (x-3)(x-3) + (y-4)(y-4) = 64$

$(x-3)^2 + (y-4)^2 = 64$

C: $(3, 4)$

r: 8