Lesson Notes 2-1 Counting Principles

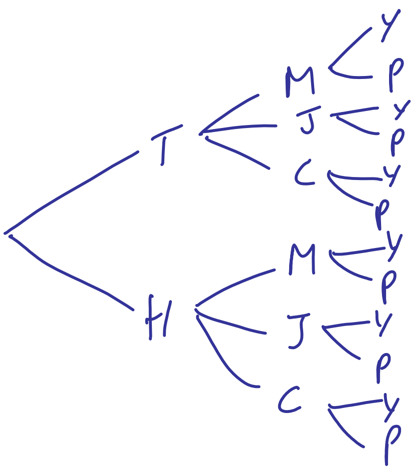
Counting was fun when you were little, but sometimes there were too many items to count. In this chapter, you will learn some short cuts to counting. In other words, you'll be counting without actually counting!

Example 1: Consider a café with a lunch special consisting of a tuna or ham sandwich (T or H); milk, juice, or coffee (M, J, C); and yogurt or pie (Y or P).

List all possible meals if you must choose one item from each of the three categories.

TMY, TMP, TJY, TJP, TCY, TCP, HMY, HMP, HJY, HJP, HCY, HCP

We can also demonstrate this using a tree diagram.



This is called the **sample space**.

How many possible meals are there in total? 12

How can you determine the number of possible meals without listing them all?

$2 \times 3 \times 2 = 12$ ← the **fundamental counting principle**

Example 2: A computer store sells 6 different computers, 4 different monitors, 5 different printers, and 3 different multimedia packages. How many different complete computer systems are available?

$6 \times 4 \times 5 \times 3 = 360$

Example 3: How many even 2 digit whole numbers are there?

$\underline{\ 9\ } \times \underline{\ 5\ } = \underline{\ 45\ }$

Example 4: In each case, how many 3 digit whole numbers can be formed using the digits 0, 2, 5, 6, 8, and 9?

a) If repetitions are allowed? $\underline{\ 5\ }\ \underline{\ 6\ }\ \underline{\ 6\ } = \underline{\ 180\ }$

b) If repetitions are not allowed? $\underline{\ 5\ } \times \underline{\ 5\ } \times \underline{\ 4\ } = \underline{\ 100\ }$

A **permutation** is an arrangement of a set of objects where the order of the objects is important. For example, given letters A, B, and C, there are different arrangements (permutations) - ABC, ACB, BAC, BCA, CAB, CBA. Note: repetition is not an option.

$\underline{\ 3\ } \times \underline{\ 2\ } \times \underline{\ 1\ } = \underline{\ 6\ }$ or ${}_3P_3 = \underline{\ 6\ }$ or $3! = \underline{3 \times 2 \times 1}$

factorial notation

Example 5: How many arrangements can be made using the letters in the word MUSIC?

$\underline{\ 5\ } \times \underline{\ 4\ } \times \underline{\ 3\ } \times \underline{\ 2\ } \times \underline{\ 1\ } = \underline{\ 120\ }$ or ${}_5P_5 = \underline{\ 120\ }$ or $\underline{\ 5\ }! = \underline{\ 120\ }$

Example 6: How many permutations can be formed using all the letters of the word CLARINET?

$8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 8! = 40320$

Example 7: How many 4 letter permutations can be formed from the letters of the word CLARINET?

$8 \times 7 \times 6 \times 5 = 1680$

Example 8: In how many ways can 1^{st}, 2^{nd}, and 3^{rd} place can be awarded in a math class of 30 students?

$30 \times 29 \times 28 = 24360$