Lesson 8: Optimization Problems

If our objective is to maximize or minimize something we can use the solution of a system of inequalities to determine this. We could substitute all x and y values into our equation and determine the highest or lowest value. The answers that give us the highest and lowest are the corners of the region shaded. The steps to for solving optimization problems are:

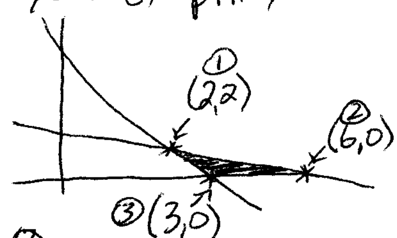
1. Read the problem, determine the unknown quantities, and assign variables.
2. From the information given, write a system of linear inequalities.
3. Determine a formula for the objective function (what we want to maximize or minimize)
4. Solve the system of linear inequalities and graph the solution region.
5. Determine the coordinates of the vertices of the solution region.
6. Determine the value of the objective function at each vertex of the solution region. Both the maximum and the minimum values of the objective function are found.

Example 1: A doctor advises a patient to take vitamin supplements to provide at least 600 mg of Vitamin A, but no more than 12 mg of iron daily. Each brand X pill contains 200 mg of Vitamin A and 2 mg of iron. Each brand Y pill contains 100 mg of Vitamin A and 4 mg of iron. Each brand X pill costs 8 cents and brand Y costs 3 cents. How many of each should he take if he wants to minimize the cost?

let x = # of pill X
y = # of pill Y

Vit A: $200x + 100y \geq 600$
Iron: $2x + 4y \leq 12$
$x \geq 0, y \geq 0$ ⇐ implicit constraints

Objective: $8x + 3y = C$



①
$C = 8(2) + 3(2)$
$\boxed{C = 22}$ $\boxed{x = 2, y = 2}$

②
$C = 8(6) + 3(0)$
$C = 48$

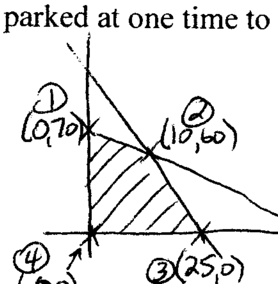
③
$C = 8(3) + 3(0)$
$C = 24$

Example 2: A parking lot has 900 m available to park vehicles. On average cars need 9 m and buses need 36 m for parking. No more than 70 vehicles can be parked in the lot at one time. The charge to park cars is \$6 a day and to park buses is \$20 a day. How many cars and how many buses should be parked at one time to produce the maximum revenue?

let x = # of buses
y = # of cars

Space: $36x + 9y \leq 900$
Number: $x + y \leq 70$
$x \geq 0, y \geq 0$

Objective: $20x + 6y = C$



①
$C = 20(0) + 6(70)$
$C = 420$

②
$\boxed{C = 20(10) + 6(60) \quad C = 560}$
$x = 10, y = 60$

③
$C = 20(0) + 6(0)$
$C = 0$

④
$C = 20(25) + 6(0)$
$C = 500$