6.3 6.4 Worksheet

0 min
0 marks

1. (a) $x = 4$ (A1)

g'' changes sign at $x = 4$ or concavity changes (R1) 2

(b) $x = 2$ (A1)

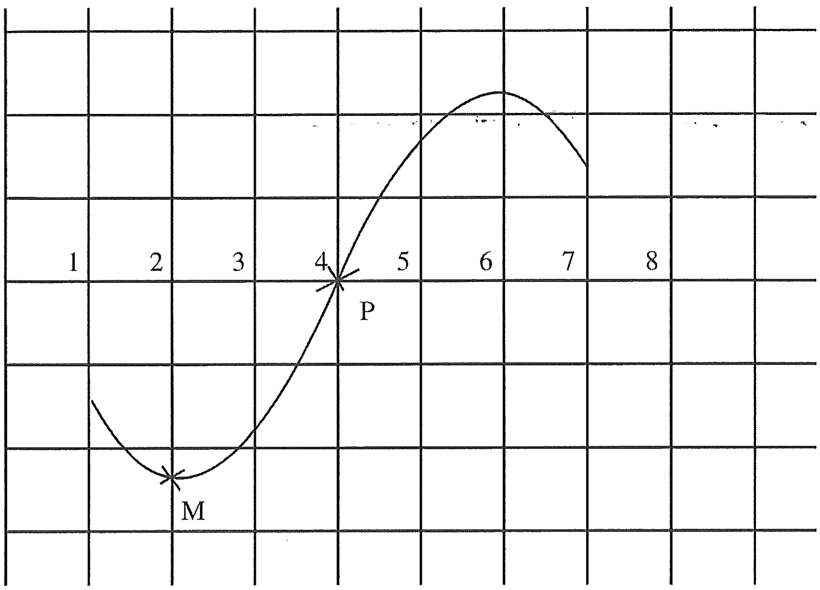
EITHER

g' goes from negative to positive (R1)

OR

$g'(2) = 0$ **and** $g''(2)$ is positive (R1) 2

(c)

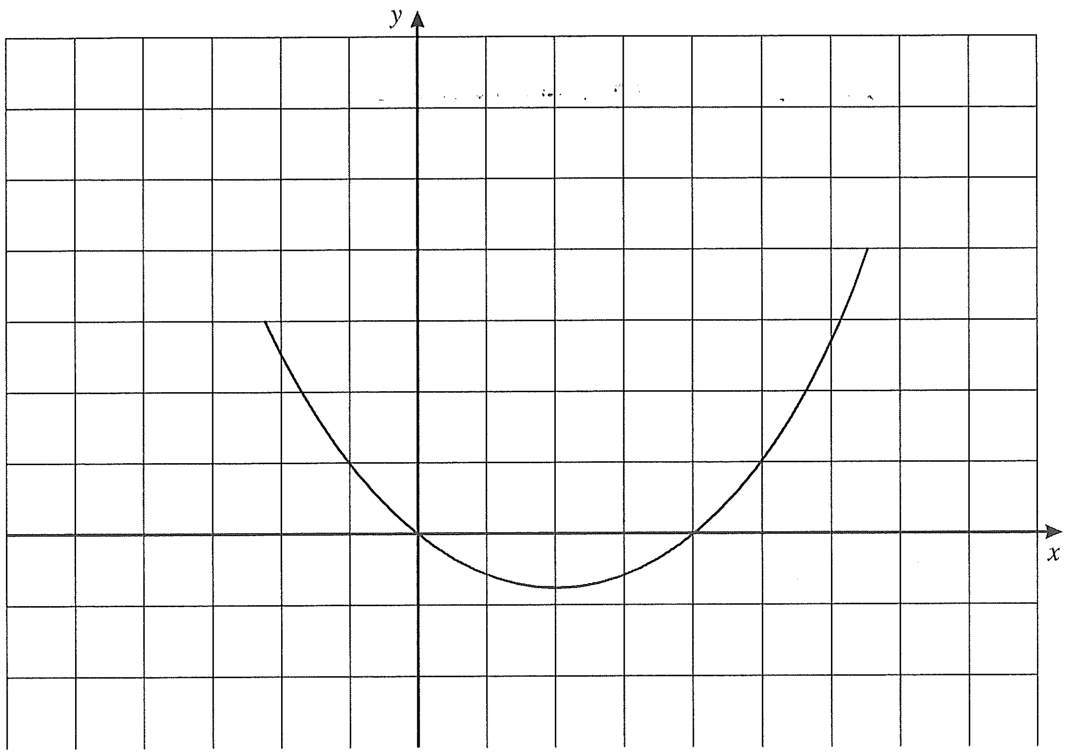


(A2)(A1)(A1) 4

Note: *Award (A2) for a suitable cubic curve through (4, 0), (A1) for M at x = 2, (A1) for P at (4, 0).*

[8]

2.



(A2)(A1)(A1)(A2) (C6)

Note: *Award A2 for correct shape (approximately parabolic), A1 A1 for intercepts at 0 and 4, A2 for minimum between* $x = 1.5$ *and* $x = 2.5$.

[6]

3. (a)

	A	B	E
$f'(x)$	negative	0	negative

A1A1A1 N3

(b)

	A	B	E
$f''(x)$	positive	positive	negative

A1A1A1 N3

[6]

4. (a) $f'(x) = 2xe^{-x} - x^2e^{-x}$ $\quad (= (2x - x^2)e^{-x} = x(2 - x)e^{-x})$ A1A1 N2

(b) Maximum occurs at $x = 2$ (A1)

Exact maximum value $= 4e^{-2}$ A1 N2

(c) For inflexion, $f''(x) = 0$ $\left((x^2 - 4x + 2) = 0,\ x = \frac{4 \pm \sqrt{16-8}}{2}, \text{etc.}\right)$ M1

$x = \frac{4+\sqrt{8}}{2}\ \left(= 2+\sqrt{2}\right)$ A1 N1

[6]

5. (a) $x = 1$ (A1) 1

(b) Using quotient rule (M1)

Substituting correctly $g'(x) = \frac{(x-1)^2(1) - (x-2)[2(x-1)]}{(x-1)^4}$ A1

$= \frac{(x-1) - (2x-4)}{(x-1)^3}$ (A1)

$= \frac{3-x}{(x-1)^3}$ (Accept $a = 3$, $n = 3$) A1 4

(c) Recognizing at point of inflexion $g''(x) = 0$ M1

$x = 4$ A1

Finding corresponding y-value $= \frac{2}{9} = 0.222$ *ie* $\mathrm{P}\left(4, \frac{2}{9}\right)$ A1 3

[8]

6. (a) $x = 1$ (A1)

EITHER

The gradient of $g(x)$ goes from positive to negative (R1)

OR

$g(x)$ goes from increasing to decreasing (R1)

OR

when $x = 1$, $g''(x)$ is negative (R1) 2

(b) $-3 < x < -2$ **and** $1 < x < 3$ (A1)

$g'(x)$ is negative (R1) 2

(c) $x = -\frac{1}{2}$ (A1)

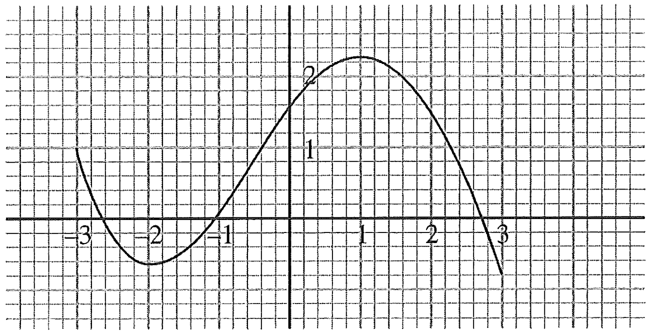
EITHER

$g''(x)$ changes from positive to negative (R1)

OR

concavity changes (R1) 2

(d)



(A3) 3

[9]

7. (a) (i) $t = 0$ $s = 800$
$t = 5$ $s = 800 + 500 - 100 = 1200$ (M1)
distance in first 5 seconds $= 1200 - 800$
$= 400$ m (A1) 2

(ii) $v = \dfrac{\mathrm{d}s}{\mathrm{d}t} = 100 - 8t$ (A1)

At $t = 5$, velocity $= 100 - 40$ (M1)
$= 60$ m s^{-1} (A1) 3

(iii) Velocity $= 36$ m s$^{-1} \Rightarrow 100 - 8t = 36$ (M1)
$t = 8$ seconds after touchdown. (A1) 2

(iv) When $t = 8$, $s = 800 + 100(8) - 4(8)^2$ (M1)
$= 800 + 800 - 256$ (A1)
$= 1344$ m (A1) 3

(b) If it touches down at P, it has 2000 – 1344 = 656 m to stop. (M1)

To come to rest, $100 - 8t = 0 \Rightarrow t = 12.5$ s (M1)

Distance covered in 12.5 $s = 100(12.5) - 4(12.5)^2$ (M1)

$= 1250 - 625$

$= 625$ (A1)

Since 625 < 656, it can stop safely. (R1) 5

[15]

8. (a) $x = 1$ (A1) 1

(b) (i) $f(-1000) = 2.01$ (A1)

(ii) $y = 2$ (A1) 2

(c) $f'(x) = \dfrac{(x-1)^2(4x-13) - 2(x-1)(2x^2-13x+20)}{(x-1)^4}$ (A1)(A1)

$= \dfrac{(4x^2-17x+13)-(4x^2-26x+40)}{(x-1)^3}$ (A1)

$= \dfrac{9x-27}{(x-1)^3}$ (AG) 3

Notes: *Award (M1) for the* ***correct*** *use of the quotient rule, the first (A1) for the placement of the correct expressions into the quotient rule.*
Award the second (A1) for doing sufficient simplification to make the given answer reasonably obvious.

(d) $f'(3) = 0$ $\Rightarrow$ stationary (or turning) point (R1)

$f''(3) = \dfrac{18}{16} > 0 \Rightarrow$ minimum (R1) 2

(e) Point of inflexion $\Rightarrow f''(x) = 0 \Rightarrow x = 4$ (A1)

$x = 4 \Rightarrow y = 0$ $\Rightarrow$ Point of inflexion = (4, 0) (A1)

OR

Point of inflexion = (4, 0) (G2) 2

[10]

9. (a) Velocity is $\frac{ds}{dt}$. (M1)

$\frac{ds}{dt} = 10 - t$ (A1)

10 (m s^{-1}) (A1) (C3)

(b) The velocity is zero when $\frac{ds}{dt} = 0$ (M1)

$10 - t = 0$

$t = 10$ (secs) (A1) (C2)

(c) $s = 50$ (metres) (A1) (C1)

Note: *Do not penalize absence of units.*

[6]