

Foundations of Math 11

Quadratic Functions & Equations

Example 1: Factor the following expressions.

a) $x^2 + 3x - 10$
 $(x+5)(x-2)$

(b) $4x^2 - 16$
 $(2x-4)(2x+4)$

(c) $2x^2 - 3x - 20$
 $x^2 - 3x - 40$
 $(x-8)(x+5)$

Example 2: Solve the following by factoring.

a) $x^3 + 7x^2 + 10x = 0$
 $x(x^2 + 7x + 10) = 0$
 $x(x+5)(x+2) = 0$
 $x=0, -5, -2$

(b) $7x^2 - 63 = -5$
 $7x^2 - 63 = 0$
 $7(x^2 - 9) = 0$
 $7(x+3)(x-3) = 0$
 $x = -3, 3$

Example 3: Solve using the quadratic formula.

a) $-3x^2 - 2x + 5 = 0$
 $x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-3)(5)}}{2(-3)}$
 $= \frac{2 \pm \sqrt{64}}{-6} = \frac{2 \pm 8}{-6} = \frac{1 \pm 4}{-3}$
 $\frac{1+4}{-3} = \frac{5}{-3}$
 $\frac{1-4}{-3} = \frac{-3}{-3} = 1$

(b) $x^2 - 4x = 1$
 $x^2 - 4x - 1 = 0$
 $x = \frac{4 \pm \sqrt{16 - 4(1)(-1)}}{2(1)}$
 $= \frac{4 \pm \sqrt{20}}{2} = \frac{4 \pm 2\sqrt{5}}{2} = 2 \pm \sqrt{5}$

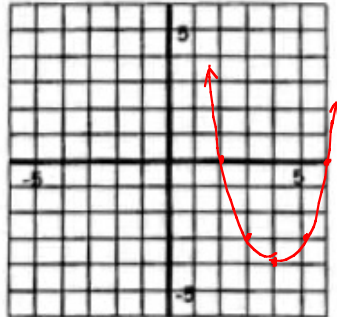
Example 4: Complete the square, state the vertex, and whether it has a maximum or minimum value for the following.

a) $y = x^2 - 6x + 10$
 $y = (x^2 - 6x + 9 - 9) + 10$
 $y = (x^2 - 6x + 9) + 10 - 9$
 $y = (x-3)^2 + 1$
 positive
 min
 $V: (3, 1)$
 min

(b) $y = \frac{1}{2}x^2 + 10x - 7$
 $y = \frac{1}{2}(x^2 + 20x + 100 - 100) - 7$
 $y = \frac{1}{2}(x+10)^2 - 7 - 50$
 $y = \frac{1}{2}(x+10)^2 - 57$
 $V: (-10, -57)$
 min

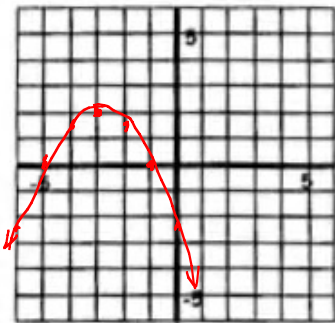
Example 5: Graph and answer the following questions.

a) $y = (x - 4)^2 - 4$



Vertex: $(4, -4)$
 Axis of symmetry: $x = 4$
 Domain: $x \in \mathbb{R}$
 Range: $y \geq -4$
 $x=0$ y-intercept: 12 $y = (0-4)^2 - 4$
 $y=0$ x-intercept: $2, 6$ $y = 12$
 $D = (x-4)^2 - 4$
 $\sqrt{4} = \sqrt{(x-4)^2}$
 $\pm 2 = x - 4$
 $4 \pm 2 = x$

b) $y = -\frac{1}{2}(x + 3)^2 + 2$



Vertex: $(-3, 2)$
 Axis of symmetry: $x = -3$
 Domain: $x \in \mathbb{R}$
 Range: $y \leq 2$
 y-intercept: -2.5
 x-intercept: $-1, -5$

Example 6: Determine the equation of the following quadratic function.

a) Has a vertex $(1, 5)$, opens down, and is congruent to $y = 2x^2$.

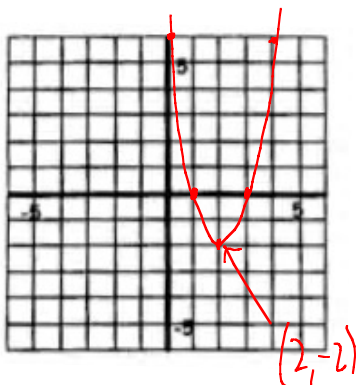
$y = -2(x - 1)^2 + 5$

b) Has a vertex $(-1, 3)$ and goes through the point $(5, 4)$.

$y = (x + 1)^2 + 3$

$4 = a(5 + 1)^2 + 3$
 $1 = a(36)$
 $\frac{1}{36} = a$
 $y = \frac{1}{36}(x + 1)^2 + 3$

c)



$y = 2(x - 2)^2 - 2$